



## Research Article

## Optimization of Gear Manufacturing for Quality and Productivity

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### ABSTRACT

Multi-objective optimization in manufacturing can effectively be solved using Multicriteria decision-making (MCDM) techniques. This paper presents the implementation methodology of the Fuzzy-MOORA hybrid technique for multi-objective optimization in laser machining of stainless-steel gears. Further, simultaneous optimization of gear quality and process productivity have been reported. Four important laser parameters, i.e., laser power, cutting speed, focal position, and gas pressure, have varied during twenty-nine experiments to machine gears by a laser process. The quality of miniature gear was measured in terms of average surface roughness, mean roughness depth, and dimensional deviation. The productivity of the laser machining process was estimated via material removal rate. An optimum set of laser machining parameters obtained after Fuzzy-MOORA optimization is laser power 2000 W, cutting speed 3 m/min, focal position -2.5 mm, and gas pressure 16 bar. This work encourages researchers and scholars to make further attempts using such MCDM techniques to develop intelligent processes in industrial and manufacturing engineering.

## INTRODUCTION

In the era of the fourth industrial revolution, manufacturing sector strives to innovate and incorporate intelligent techniques for product manufacturing. To stay competitive, industries are increasing capacities and adopting the best machineries, tools, equipment, and processes. Good product design, efficient plant layout, low manufacturing cost and time, quick response etc. are some of the attributes of the modern manufacturing industry. All activities including manufacturing processes require systematic planning and optimization. Automation, optimization, machine learning based fault detection, and intelligent maintenance etc. are key techniques [1].

Cost, quality, and productivity are important factors and need to be optimized for the overall success in manufacturing. Engineered products and mechanical components such as gears are backbone of many industries [2]. Manufacturing of high quality gears is therefore essential for the better performance of the machineries, devices, and instruments where gears are used. The commercial manufacturing of gears is accomplished with the help of many types of manufacturing processes including machining [2]. Advanced machining processes such as electric discharge machining, laser cutting, and abrasive water jet machining etc. are being attempted as alternate of conventional manufacturing to develop commercial gears [3,4]. In all of these

processes, gear quality and process productivity are considered as major responses or outputs. Both responses are important but conflicting in nature as it needs a compromise with quality when the objective is to achieve high productivity. It is because the values of parameters required to increase process speed or material removal rate to achieve high productivity mostly produce low manufacturing quality [5-7]. This compels to apply a suitable optimization technique for simultaneous optimization of process parameters to secure the best values of quality and productivity at a single set of parameters. Multi-criteria decision making (MCDM) optimization techniques such as Multi-Objective Optimization Based on Ratio Analysis (MOORA), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR), and Analytical hierarchy (AHP) etc., have been found very effective for the same problems in various manufacturing processes [5]. Hybridization of these processes with soft computing or evolutionary algorithms, increases the effectiveness. Fuzzy-MOORA is one of such hybrid technique which has been found useful by many past researchers. Some important past research attempts on Fuzzy-MOORA based optimization of manufacturing processes are discussed below.

In a recent study, Jhodkar et al. [8] successfully utilized Fuzzy-MOORA for multi-objective optimization in turning of Titanium. Machining at optimum combination of process parameters i.e., cutting speed 140 m/min, tool overhang length 65 mm, and

hardness of work material 1934 HV, resulted in the best machinability in terms of the optimum values of tool flank wear, cutting force, and surface roughness, simultaneously. Emovon et al. [9] conducted an investigation on optimization in design and fabrication of an automated hammering machine and found Fuzzy-MOORA effective indeed and at par with other Fuzzy based hybrid techniques. To simultaneously optimize two important machinability indicators, namely, surface roughness and micro-hardness of nitinol work material, Majumder and Maity [10] conducted a multiperformance optimization of wire-EDM process to yield the best results by employing Fuzzy-MOORA hybrid technique. Nitinol is a type of shape memory alloy and while machining this material by advanced processes such as wire-EDM, it's a normal to encounter with the multicriteria decision making problems. It is imperative to make a selection of the suitable optimization technique. With a hybridization of Fuzzy and MOORA, the optimum wire-EDM parameters for improvement in roughness and hardness of the machined shape memory alloy can be secured. Not only for manufacturing, the hybrid technique of Fuzzy-MOORA has been found effective to solve complex decision-making problems in industrial engineering scenario [11,12].

Gupta and Jain [6] used desirability analysis for multi-performance optimization during wire electric discharge machining of gears [6]. They obtained manufacturing quality of German standard DIN 7 and 1.1-microns surface finish. Anghel et al. [3,13] also use desirability analysis technique for simultaneous optimization of gear quality and productivity of the laser machining process. They considered mean roughness depth, average roughness, dimensional deviation, and material removal rate, as machinability indicators to estimate quality and productivity respectively. Optimum values of four important laser parameters, namely, laser power, focal distance, cutting speed, and gas pressure, resulted in miniature stainless-steel gears equipped with average roughness Ra- 0.43 microns. Additionally, a better surface quality in terms of thinner heat affected zone was obtained. Multi-response optimization successfully improved gear surface quality and laser productivity simultaneously with not more than 6% difference between predicted and experimental values. Phokane et al. [14] used particle swarm optimization and obtained the best set of abrasive water jet machining parameters for improvement in mean roughness depth of miniature gears of brass. It was a single-performance or single-objective optimization attempt and resulted in 5.85% improvement in the value of mean roughness depth (Rz- 4.10  $\mu\text{m}$ ). Although they considered some responses in their work for quality and productivity, but no multi-response-based optimization was reported.

A review of the past work on optimization of manufacturing processes using Fuzzy-MOORA indicates that this hybrid technique has a potential to perform multi-objective optimization for improvement in quality, productivity, and other outputs. A review of the past work on multi-objective optimization of gear machining processes, reveals that most of the work is based on traditional statistical techniques and no work is found on Fuzzy based or any other soft computing technique. The work reported in this paper attempts to fill this gap with the following objectives:

- To optimize the laser machining parameters and secure their best values for conflicting responses i.e., quality (surface

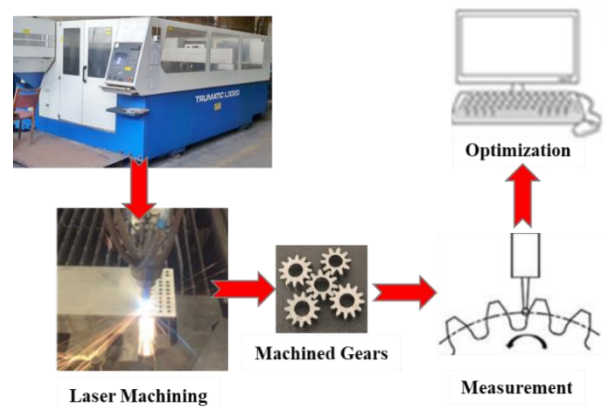


Figure 1. Sequence of tasks in laser machining and optimization of gears

roughness and dimensional deviation) and productivity (MRR).

- To hybridize two techniques i.e., fuzzy and MOORA and harnessing their potential for parametric optimization in laser machining of gears.
- To set appropriate criteria and weights for a better performance of optimization techniques.
- To normalize the experimental data for a better synchronization and enhanced accuracy of the optimization results.
- To make recommendations on the effectiveness of Fuzzy-MOORA hybrid technique for gear manufacturing with possible optimum outputs.

In this paper, we have reported the important aspect of the optimization of laser cutting parameters for manufacturing of quality gears. A detailed methodology of implementation of Fuzzy-MOORA hybrid technique for simultaneous optimization of gear quality and productivity of the laser cutting process and outcomes are discussed.

## METHOD

Figure 1 presents the steps followed for manufacturing of gears by laser machining and its optimization. Table 1 shows the representation of four controllable factors, namely, laser power, cutting speed, focal position, and gap pressure together with their high, medium and low levels as reported in [3,4]. To design the experiments using four factors, where each of them is varied at three levels, Box Behnken technique of response surface methodology has been used [15]. Twenty-nine experimental combinations or settings of laser machining parameters have been generated using their values and levels as given in Table 1.

Table 1: Variable Parameters and Their Levels for Laser Manufacturing of Gears [4]

Variable Parameter	Unit	Levels and Corresponding Values		
		1	2	3
Laser Power	W	1500	2000	2500
Cutting Speed	m/min	1	2	3
Focal Position	mm	-3.5	-2.5	-1.5
Gas Pressure	bar	10	13	16

Therefore, a total of twenty-nine experiments have been conducted and the values of response parameters, namely, average roughness, mean roughness depth, dimensional deviation; and material removal rate, have been measured/obtained against every single experiment.

MOORA method can be utilized to determine the most suitable alternative combinations for multi-criteria decision making. Brauers and Zavadskas founded [16] this methodology while attempting to optimize numerous conflicting criteria that were subject to several constraints. The two most vital factors in the MOORA method are the ratio system and the reference point, where each alternative's overall performance is determined. Manufacturing plants, insurance, industrial and banking sectors are some of the areas in which this method is used, wherein some of these fields encounter issues where more than two elements conflict each other and the best choice needs to be identified. Such cases are optimized by utilizing the MOORA techniques and many researchers have used this technique so far [16].

Step 1: Equation 1 makes use of the input parameters to give performance of responses as initiated by the MOORA method.

$$P = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1b} \\ p_{21} & p_{22} & \dots & p_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ p_{a1} & p_{a2} & \dots & p_{ab} \end{bmatrix} \quad (1)$$

Nomenclature for where the  $p_{ij}$  are the response outputs of the  $i^{th}$  alternative on the  $j^{th}$  criterion, the number of several criteria and alternatives are represented by  $a$  and  $b$ .

Step 2: Equation 2 below uses the formation ration system to normalize the data decision matrix.

$$p_{ij}^* = \frac{p_{ij}}{\sqrt{\sum_{i=1}^b p_{ij}^2}} \quad (j = 1, 2, \dots, n) \quad (2)$$

where the  $p^*_{ij}$  would be the normalized value that is in between 0 and 1, it is also a dimensionless quantity of the  $i^{th}$  alternative on the  $j^{th}$  criterion.

Step 3: MOORA index is used to identify the ranking scores. Alternatively, Equation 3 is employed in subtracting and adding of weighted normalized values in line with each alternative to determine the overall assessment values ( $q_i$ ). Normalized values in minimization of non-beneficial (lower-is-better) are subtracted wherein the case maximization of beneficial (higher-is-better) response are added to obtain the overall assessment values in multi-objective optimization.

$$q_i = \sum_{j=1}^x p_{ij}^* - \sum_{j=x+1}^y p_{ij}^* \quad (3)$$

where the  $x$  is the number of criteria that must be maximized belonging to the benefit responses,  $y$  represents the number of criteria that requires to be minimized.  $q_i$ , depending on all criteria, denotes the assessed normalized number of the  $i^{th}$  alternative.

It was initially thought that some criteria were more important than others and by assigning additional weight, it can be accomplished. In such a case, Equation 4 would be the better representation of Equation 3.

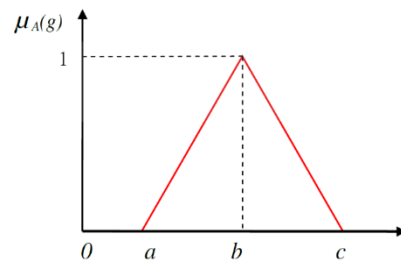


Figure 2. A Triangular Fuzzy Membership Function

$$q_i = \sum_{j=1}^x r_j p_{ij}^* - \sum_{j=x+1}^y r_j p_{ij}^* \quad (4)$$

where  $r_j$  is the weight of the  $j^{th}$  criteria.

Step 4: The overall computed assessment values in the decision matrix are used to determine either negative or positive based on non-beneficial or beneficial factors respectively. The MOORA value  $q_i$  finds the optimal result value which illustrates both the lowest value (worst result) and highest value (best result).

To ensure that best outcomes are obtained for multi conflicting criteria-based manufacturing, the fuzzy set theory assists in the treatment of uncertainties in the ambiguity and vagueness form. The linguistic approach was constructed by fuzzy logic, in the fuzzy set theory, whereby the variables can assume values that are linguistic [17]. The given options by the decision makers are termed as quantified linguistic variables, with the assistance of the fuzzy set theory. For converting aforementioned variables that are linguistic into a different fuzzy number, a fuzzy membership function is used. Figure 2 illustrates the Fuzzy membership function in a triangular form.

There are essential fuzzy set theory and fuzzy number definitions are explained as follows [18-19]:

Definition 1: The grade of membership of  $g$  in  $\tilde{P}$  represents the fuzzy set  $\tilde{P}$  in a universe of discourse  $X$  as explained by a membership function  $\mu_{\tilde{A}}(g)$ .

Definition 2:  $\tilde{P}$  represents the membership function of the fuzzy value which is determined by Equation 5, where  $\tilde{P} = (p_1, p_2, p_3)$  represents the triangular fuzzy numbers (TFNs).

$$\mu_{\tilde{A}}(g) = \begin{cases} 0 & g < p_1 \\ \frac{g-p_1}{p_2-p_1} & p_1 \leq g \leq p_2 \\ \frac{p_3-g}{p_3-p_2} & p_2 \leq g \leq p_3 \\ 0 & g > p_3 \end{cases} \quad (5)$$

Definition 3: Triangular fuzzy numbers can be given by the fuzzy subtraction and fuzzy sum of 2 different triangular fuzzy numbers. However, the multiple of 2 different triangular fuzzy numbers yields an approximate TFN. For instance, the following 2 TFNs  $\tilde{P} = (p_1, p_2, p_3)$  and  $\tilde{Q} = (q_1, q_2, q_3)$  with a positive real number  $w = (w, w, w)$ , a number of vital operations of fuzzy numbers can be represented by Equations 6 -10 as follow:

$$\tilde{P}(+) \tilde{Q} = (p_1 + p_2, q_1 + p_2, q_1 + r_2) \quad (6)$$

$$\tilde{P}(-)\tilde{Q} = (p_1 - p_2, q_1 - q_2, r_1 - r_2) \tag{7}$$

$$\tilde{P}(\times)\tilde{Q} = (p_1p_2, q_1q_2, r_1r_2) \tag{8}$$

$$\tilde{P}(/)\tilde{Q} = (p_1/q_1, p_2/q_2, p_3/p_3) \tag{9}$$

$$\tilde{P}(\times)w = (p_1w, p_2w, p_3w) \tag{10}$$

Definition 4: A TFN  $\tilde{P} = (p_1, p_2, p_3)$ , Equation 11 determines the defuzzified value  $\alpha(\tilde{P})$ :

$$\alpha(\tilde{P}) = \frac{p_1+p_2+p_3}{3} \tag{11}$$

Definition 5: The distance between  $\tilde{P}$  and  $\tilde{Q}$  for TFNs  $\tilde{P} = (p_1, p_2, p_3)$ , and  $\tilde{Q} = (q_1, q_2, q_3)$  is calculated as follows:

$$(\tilde{P}, \tilde{Q}) = \sqrt{\frac{1}{3}(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2} \tag{12}$$

Definition 6: Equation 13 finds the best non-fuzzy performance (BNP) number through the center of area approach:

$$NP_i = \frac{[(r-p)+(q-p)]}{3} + p, \forall_i \tag{13}$$

In the hybrid fuzzy-MOORA technique, the options of decision makers are conveyed in relation to linguistic variables set [4, 9]. The following steps define the fuzzy embedded MOORA technique:

Step 1: Equation 14 creates the fuzzy decision matrix amongst all criteria and alternatives that belong to the TFNs.

$$\tilde{P} = \begin{bmatrix} [p_{11}^a, p_{11}^b, p_{11}^c] & [p_{12}^a, p_{12}^b, p_{12}^c] & [p_{1c}^a, p_{1c}^b, p_{1c}^c] \\ \dots & \dots & \dots \\ [p_{b1}^a, p_{b1}^b, p_{b1}^c] & [p_{b2}^a, p_{b2}^b, p_{b2}^c] & [p_{bc}^a, p_{bc}^b, p_{bc}^c] \end{bmatrix} \tag{14}$$

Step 2: The normalized fuzzy decision matrix can be computed using Equations 15 to 17.

$$p_{ij}^{a*} = \frac{p_{ij}^a}{\sqrt{\sum_{i=1}^b [(p_{ij}^a)^2 + (p_{ij}^b)^2 + (p_{ij}^c)^2]}} \tag{15}$$

$$p_{ij}^{b*} = \frac{p_{ij}^b}{\sqrt{\sum_{i=1}^m [(p_{ij}^a)^2 + (p_{ij}^b)^2 + (p_{ij}^c)^2]}} \tag{16}$$

$$p_{ij}^{c*} = \frac{p_{ij}^c}{\sqrt{\sum_{i=1}^b [(p_{ij}^a)^2 + (p_{ij}^b)^2 + (p_{ij}^c)^2]}} \tag{17}$$

Step 3: Equations 18 to 20 then determine the normalized fuzzy decision matrix.

$$W_{ij}^a = r_j p_{ij}^{a*} \tag{18}$$

$$W_{ij}^b = r_j p_{ij}^{b*} \tag{19}$$

$$W_{ij}^c = r_j p_{ij}^{c*} \tag{20}$$

Step 4: The overall fuzzy assessment number  $\tilde{q}_i$  can be converted to the non-fuzzy value (crisp) then compute the best non-fuzzy performance (BNP) using Equation 21 as follows:

$$BNP_i(\tilde{q}_i) = \frac{(q_i^c - q_i^a) + (q_i^b - q_i^a)}{3} + q_i^a \tag{21}$$

where  $\tilde{q}_i = (q_i^a, q_i^b, q_i^c)$

Step 5: Equation 22 can be applied to calculate the overall fuzzy assessment value in this step.

$$\tilde{q}_i = \tilde{W}_{ij}^+ - \tilde{W}_{ij}^- \tag{22}$$

Where  $\tilde{W}_{ij}^+$  and  $\tilde{W}_{ij}^-$  represent the overall assessment value of the beneficial and non-beneficial criteria respectively.

Step 6: Then, in descending order, give ranking to all the calculated closeness values wherein the worst alternative refer by least closeness value that shows the worst performance and vice versa.

## RESULT AND DISCUSSION

Table 2 represents the experimental layout for laser gear cutting together with values of gear quality indicators i.e. mean roughness depth, average surface roughness, dimensional deviation, and process productivity indicator i.e. material removal rate.

Successful application of the Fuzzy-MOORA technique is generally achieved by first applying Fuzzy, this process is called fuzzification of alternatives then applying the MOORA technique in the final step. To begin the fuzzification process, Table 3 defining the linguistic variables used for each criterion is adopted for the conversion of the crisp value responses into fuzzy numbers (fuzzy set theory) using triangular fuzzy numbers (TFN's).

In this study, each alternative was mainly identified in terms of specific linguistic variables as illustrated in Table 3. This was done to calculate the weights of the selected output criteria, namely – Rz, Ra, MRR and DD respectively, as shown in Table 4. The rule of thumb is that linguistic variable used for each criterion ranges from 0 to 1 and the incremental change from one alternative to the next is with an odd number. In this application, only seven linguistic variables are used. In the selection of the best criteria to make a decision, the relative weight of each criterion is chosen from the linguistic variable used for each criterion (Table 3), that is it can range from very low (VL) to very high (VH). All criteria, i.e. Rz, Ra, DD and MRR are given a “VH” weight by the decision maker in this study as show in Table 4. The minimization of Rz, Ra and DD is equally important as the maximization of MRR.

The linguistic variable used for each criterion (Table 3) conversion into linguistic variable used for each alternative (Table 5) is done by multiplying the triangular fuzzy number's (TFN's) by 10 to have them as whole numbers. It is also important to note that the linguistic variables also change, i.e., very low (VL) changed to very poor (VP) and so on.

Table 6 shows the conversion of the experimental data alternatives into linguistic variables. This was done by taking the difference between maximum and minimum values and dividing by 7 (number of linguistic variables) to find a value that is added to the minimum number 7 times to arrive at the maximum number. The 7 ranges of data sets were then assigned the

Table 1: Experimental Results Corresponding to All Set of Parameters for Laser Machining of Gears [4]

Variable Input Parameters					Experimental Responses			
Run No.	Laser Power (W)	Cutting Speed (m/min)	Focal Position (mm)	Gas Pressure (bar)	Mean Roughness Depth "Rz" values ( $\mu\text{m}$ )	Average Roughness "Ra" values ( $\mu\text{m}$ )	Material Removal Rate "MRR" values ( $\text{mm}^3/\text{min}$ )	Dimensional Deviation "DD" values (%)
1	1500	1	-2.5	13	3.69	0.74	2020.3	1.0264
2	2500	1	-2.5	13	2.21	0.43	2367.81	1.0268
3	1500	3	-2.5	13	2.36	0.47	5353.33	1.5244
4	2500	3	-2.5	13	3.4	0.76	6189.06	0.9088
5	2000	2	-3.5	10	4.74	0.93	4539.1	1.6582
6	2000	2	-1.5	10	4.33	0.89	3310.71	1.2305
7	2000	2	-3.5	16	4.85	0.96	4283.61	1.0342
8	2000	2	-1.5	16	3.42	0.67	4538.32	1.5174
9	1500	2	-2.5	10	3.15	0.59	3241.24	1.3131
10	2500	2	-2.5	10	3.17	0.65	4502.19	0.9945
11	1500	2	-2.5	16	3.31	0.68	4184.55	1.3074
12	2500	2	-2.5	16	2.53	0.49	4519.06	0.7351
13	2000	1	-3.5	13	4.98	1.03	2338.04	1.0174
14	2000	3	-3.5	13	3.87	0.77	6340.24	1.3445
15	2000	1	-1.5	13	2.63	0.54	2070.51	1.3494
16	2000	3	-1.5	13	3.89	0.82	6121.94	1.4065
17	1500	2	-3.5	13	3.21	0.66	4044.73	1.2608
18	2500	2	-3.5	13	4.46	0.86	4288.95	1.3899
19	1500	2	-1.5	13	3.64	0.73	3879.66	1.9142
20	2500	2	-1.5	13	2.23	0.45	4180.18	0.9525
21	2000	1	-2.5	10	2.79	0.57	2097.63	0.9588
22	2000	3	-2.5	10	4.85	0.97	6204.26	1.2931
23	2000	1	-2.5	16	4.27	0.89	2078.93	0.9925
24	2000	3	-2.5	16	2.52	0.51	6355.23	0.8919
25	2000	2	-2.5	13	2.73	0.56	4010.75	1.0574
26	2000	2	-2.5	13	2.83	0.58	4071.45	0.9831
27	2000	2	-2.5	13	2.87	0.59	3814.42	1.0582
28	2000	2	-2.5	13	2.97	0.61	4092.38	1.0636
29	2000	2	-2.5	13	2.82	0.58	3983.22	1.0131

Table 3. Linguistic Variable Used for Each Criterion

Linguistic Variable	Triangular Fuzzy Number (TFNs)
Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

Table 4. Relative Weight of Each Criterion

Relative Weight of Each Criteria		
Criteria	Decision Maker	Fuzzy Number
Rz	VH	(0.9, 1.0, 1.0)
Ra	VH	(0.9, 1.0, 1.0)
MRR	VH	(0.9, 1.0, 1.0)
DD	VH	(0.9, 1.0, 1.0)

Table 5. Linguistic Variable Used for Each Alternative

Linguistic Variable	Triangular Fuzzy Number (TFNs)
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium Poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

linguistic variables: "VG" for the first range, "G" for the second range and so on up "VP" for the last range.

Furthermore, the fuzzy decision matrix compilation was achieved through the conversion of the data sets calculated after the aforementioned assessment process into the correct triangular fuzzy numbers, these results of the conversion process are illustrated in Table 7. This is achieved by substituting all the

Table 6. Fuzzy Linguistic Variables

Experimental Responses					Fuzzy Linguistic Variables			
Run	Rz (µm)	Ra (µm)	MRR (mm <sup>3</sup> /min)	DD (%)	Rz (µm)	Ra (µm)	MRR (mm <sup>3</sup> /min)	DD (%)
1	3.69	0.74	2020.3	1.0264	F	F	VP	G
2	2.21	0.43	2367.81	1.0268	VG	VG	VP	G
3	2.36	0.47	5353.33	1.5244	G	VG	G	MP
4	3.4	0.76	6189.06	0.9088	MG	F	VG	G
5	4.74	0.93	4539.1	1.6582	VP	P	MG	VP
6	4.33	0.89	3310.71	1.2305	P	P	MP	MG
7	4.85	0.96	4283.61	1.0342	VP	VP	F	G
8	3.42	0.67	4538.32	1.5174	F	MG	MG	MP
9	3.15	0.59	3241.24	1.3131	MG	G	P	F
10	3.17	0.65	4502.19	0.9945	MG	MG	MG	G
11	3.31	0.68	4184.55	1.3074	MG	MG	F	F
12	2.53	0.49	4519.06	0.7351	G	VG	MG	VG
13	4.98	1.03	2338.04	1.0174	VP	VP	VP	G
14	3.87	0.77	6340.24	1.3445	MP	F	VG	F
15	2.63	0.54	2070.51	1.3494	G	G	VP	F
16	3.89	0.82	6121.94	1.4065	MP	MP	VG	F
17	3.21	0.66	4044.73	1.2608	MG	MG	F	F
18	4.46	0.86	4288.95	1.3899	P	MP	F	F
19	3.64	0.73	3879.66	1.9142	F	F	F	VP
20	2.23	0.45	4180.18	0.9525	G	VG	F	G
21	2.79	0.57	2097.63	0.9588	G	G	VP	G
22	4.85	0.97	6204.26	1.2931	VP	VP	VG	F
23	4.27	0.89	2078.93	0.9925	P	P	VP	G
24	2.52	0.51	6355.23	0.8919	G	VG	VP	VG
25	2.73	0.56	4010.75	1.0574	G	G	G	G
26	2.83	0.58	4071.45	0.9831	G	G	VG	G
27	2.87	0.59	3814.42	1.0582	G	G	MG	G
28	2.97	0.61	4092.38	1.0636	G	MG	MP	G
29	2.82	0.58	3983.22	1.0131	G	G	F	G

Table 7. Fuzzy Decision Matrix

Run	Rz (µm)	Ra (µm)	MRR (mm <sup>3</sup> /min)	DD (%)	Run	Rz (µm)	Ra (µm)	MRR (mm <sup>3</sup> /min)	DD (%)
1	(3, 5, 7)	(3, 5, 7)	(0, 0, 1)	(7, 9, 10)	16	(1, 3, 5)	(1, 3, 5)	(9, 10, 10)	(3, 5, 7)
2	(9, 10, 10)	(9, 10, 10)	(0, 0, 1)	(7, 9, 10)	17	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
3	(7, 9, 10)	(9, 10, 10)	(7, 9, 10)	(1, 3, 5)	18	(0, 1, 3)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)
4	(5, 7, 9)	(3, 5, 7)	(9, 10, 10)	(7, 9, 10)	19	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(0, 0, 1)
5	(0, 0, 1)	(0, 1, 3)	(5, 7, 9)	(0, 0, 1)	20	(7, 9, 10)	(9, 10, 10)	(3, 5, 7)	(7, 9, 10)
6	(0, 1, 3)	(0, 1, 3)	(1, 3, 5)	(5, 7, 9)	21	(7, 9, 10)	(7, 9, 10)	(0, 0, 1)	(7, 9, 10)
7	(0, 0, 1)	(0, 0, 1)	(3, 5, 7)	(7, 9, 10)	22	(0, 0, 1)	(0, 0, 1)	(9, 10, 10)	(3, 5, 7)
8	(3, 5, 7)	(5, 7, 9)	(5, 7, 9)	(1, 3, 5)	23	(0, 1, 3)	(0, 1, 3)	(0, 0, 1)	(7, 9, 10)
9	(5, 7, 9)	(7, 9, 10)	(0, 1, 3)	(3, 5, 7)	24	(7, 9, 10)	(9, 10, 10)	(9, 10, 10)	(9, 10, 10)
10	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(7, 9, 10)	25	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)	(7, 9, 10)
11	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)	26	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)	(7, 9, 10)
12	(7, 9, 10)	(9, 10, 10)	(5, 7, 9)	(9, 10, 10)	27	(7, 9, 10)	(7, 9, 10)	(1, 3, 5)	(7, 9, 10)
13	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	(7, 9, 10)	28	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)
14	(1, 3, 5)	(3, 5, 7)	(9, 10, 10)	(3, 5, 7)	29	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)	(7, 9, 10)
15	(7, 9, 10)	(7, 9, 10)	(0, 0, 1)	(3, 5, 7)					

values of the linguistic variables by the triangular fuzzy numbers (TFN). The fuzzy decision matrix shown in Table 7, illustrates the normalization of the data sets and the outcomes are illustrated in Table 8. The weighted normalized fuzzy decision matrix was obtained by multiplying the specified weights of each criterion with their corresponding values.

The data sets given in Table 8 were then converted into crisp value from the application of Equation 21 and shown in Table 9

to complete the application of Fuzzy. MOORA is applied to give the overall assessment values were computed using Equation 22 and listed in Table 10.

As per Table 10, based on the overall assessment value, Experiment number 24 has secured the first rank, that indicates Fuzzy-MOORA based optimum parameters for laser machining of gear with the best values of gear quality and process productivity indicators.

Table 8. Product of Relative Weights of Each Criterion and Normalized Fuzzy Decision Matrix

Run	Rz ( $\mu\text{m}$ )	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{min}$ )	DD (%)	Run	Rz ( $\mu\text{m}$ )	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{min}$ )	DD (%)
1	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	16	(0.09, 0.3, 0.5)	(0.09, 0.3, 0.5)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)
2	(0.81, 1, 1)	(0.81, 1, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	17	(0.45, 0.7, 0.9)	(0.45, 0.7, 0.9)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)
3	(0.63, 0.9, 1)	(0.81, 1, 1)	(0.09, 0.3, 0.5)	(0.09, 0.3, 0.5)	18	(0, 0.1, 0.3)	(0.09, 0.3, 0.5)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)
4	(0.45, 0.7, 0.9)	(0.27, 0.5, 0.7)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	19	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	(0, 0, 0.1)	(0, 0, 0.1)
5	(0, 0, 0.1)	(0, 0.1, 0.3)	(0, 0, 0.1)	(0, 0, 0.1)	20	(0.63, 0.9, 1)	(0.81, 1, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
6	(0, 0.1, 0.3)	(0, 0.1, 0.3)	(0.45, 0.7, 0.9)	(0.45, 0.7, 0.9)	21	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
7	(0, 0, 0.1)	(0, 0, 0.1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	22	(0, 0, 0.1)	(0, 0, 0.1)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)
8	(0.27, 0.5, 0.7)	(0.45, 0.7, 0.9)	(0.09, 0.3, 0.5)	(0.09, 0.3, 0.5)	23	(0, 0.1, 0.3)	(0, 0.1, 0.3)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
9	(0.45, 0.7, 0.9)	(0.63, 0.9, 1)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	24	(0.63, 0.9, 1)	(0.81, 1, 1)	(0.81, 1, 1)	(0.81, 1, 1)
10	(0.45, 0.7, 0.9)	(0.45, 0.7, 0.9)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	25	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
11	(0.45, 0.7, 0.9)	(0.45, 0.7, 0.9)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	26	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
12	(0.63, 0.9, 1)	(0.81, 1, 1)	(0.81, 1, 1)	(0.81, 1, 1)	27	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
13	(0, 0, 0.1)	(0, 0, 0.1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	28	(0.63, 0.9, 1)	(0.45, 0.7, 0.9)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
14	(0.09, 0.3, 0.5)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)	29	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.63, 0.9, 1)
15	(0.63, 0.9, 1)	(0.63, 0.9, 1)	(0.27, 0.5, 0.7)	(0.27, 0.5, 0.7)					

Table 9. Crisp Values for Weighted Normalized Fuzzy Decision Matrix

Run	Rz ( $\mu\text{m}$ )	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{min}$ )	DD (%)	Run	Rz ( $\mu\text{m}$ )	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{min}$ )	DD (%)
1	0.490	0.490	0.033	0.843	16	0.297	0.297	0.937	0.49
2	0.937	0.937	0.033	0.843	17	0.683	0.683	0.49	0.49
3	0.843	0.937	0.843	0.297	18	0.133	0.297	0.49	0.49
4	0.683	0.49	0.937	0.843	19	0.49	0.49	0.49	0.033
5	0.033	0.133	0.683	0.033	20	0.843	0.937	0.49	0.843
6	0.133	0.133	0.297	0.683	21	0.843	0.843	0.033	0.843
7	0.033	0.033	0.49	0.843	22	0.033	0.033	0.937	0.49
8	0.49	0.683	0.683	0.297	23	0.133	0.133	0.033	0.843
9	0.683	0.843	0.133	0.49	24	0.843	0.937	0.937	0.937
10	0.683	0.683	0.683	0.843	25	0.843	0.843	0.49	0.843
11	0.683	0.683	0.49	0.49	26	0.843	0.843	0.49	0.843
12	0.843	0.937	0.683	0.937	27	0.843	0.843	0.297	0.843
13	0.033	0.033	0.033	0.843	28	0.843	0.683	0.49	0.843
14	0.297	0.49	0.937	0.49	29	0.843	0.843	0.49	0.843
15	0.843	0.843	0.033	0.49					

Table 10. MOORA Application: Overall Assessment Value

Run	$Y_i = Rz + Ra + MRR + DD$	Rank	Run	$Y_i = Rz + Ra + MRR + DD$	Rank
1	1.857	21	16	2.02	20
2	2.75	12	17	2.347	14
3	2.92	8	18	1.41	24
4	2.953	7	19	1.503	22
5	0.883	29	20	3.113	3
6	1.247	26	21	2.563	13
7	1.4	25	22	1.493	23
8	2.153	18	23	1.143	27
9	2.15	19	<b>24</b>	<b>3.653</b>	<b>1</b>
10	2.893	9	25	3.02	4
11	2.347	14	26	3.02	4
12	3.4	2	27	2.827	11
13	0.943	28	28	2.86	10
14	2.213	16	29	3.02	4
15	2.21	17			

## CONCLUSION

This paper has reported an implementation of Fuzzy-MOORA as a solution to multicriteria decision making (MCDM) problem

related to simultaneous optimization of quality and productivity during laser machining of miniature gears. Fuzzy-MOORA integrated hybrid optimization resulted in optimum values of laser machining parameters i.e. laser power 2000 W, cutting

speed 3 m/min, focal position -2.5 mm, and gas pressure 16 bar for a ready industrial reference to obtain the best values of surface quality of miniature gears and productivity of the process. The future research directions include hybridizing Fuzzy with other suitable optimization technique for laser machining of gears, comparing Fuzzy based hybrid optimization techniques, and using other hybrid and soft computing techniques for optimization in gear manufacturing processes. It is hoped that the details of Fuzzy-MOORA implementation i.e. methodology followed and results obtained in this work, will facilitate the researchers to solve complex decision-making problems for a wide range of manufacturing processes and systems.

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